Modern Sectors vs. Traditional Sectors and Economic Growth

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Abstract

This paper presents a theory of how sectors are endogenously created in an economy and what affects their number or distribution. More specifically it examines how growth and development affect the distribution of sectors. The main result of the paper is that, as the economy develops, initially sectors become less concentrated, namely economic activity is spread across a larger variety of sectors but there exists a level of development beyond which sectors begin to concentrate again across smaller variety of sectors. In other words, the number of sectors follows an inverse U shaped curve. Another element of the paper is the distinction between modern and traditional sectors. The paper discusses the co-evolution of these two types of sectors, and the feedback effect from one type to the other.

Keywords: Traditional sectors, Modern sectors, Diversification, Economic structure.

JEL Classifications: L11, L16, L22, O11, O41.

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1 Introduction

Recent decades have seen a dramatic increase in the literature on economic growth and development. Naturally, this literature is mainly macroeconomic and focuses on how aggregate productivity of countries rises. But economists have always known that the process of economic growth entails deep structural changes in the economy. The obvious one is the move from Agriculture to Industry and later from Industry to Services. But the process of development is deeper than that and it involves a deep change in the goods produced and consumed in the economy, and thus it involves continuous change in the sectors of the growing economy. This issue has not been studied sufficiently and it is to this topic that we turn in this paper. We try to build a simple theory of sectors in order to analyze how they evolve during the process of economic growth. Our research is motivated mainly by the empirical findings on the sector diversification along the process of growth.

When considering development as transition from agriculture to industry and then to services, one is naturally led to think of a process of growing labor specialization and hence of moving to more and more sectors. Accordingly, more developed economies are expected to have larger number of sectors. Interestingly, when the evolution of sector concentration is empirically studied in Imbs and Wacziarg (2003), the pattern found is strikingly different. This paper provides new and robust evidence that economies grow through two stages of diversifications. At first, diversification to sectors increases, but there is a level of per capita income, beyond which the sector distribution of economic activity begins to con-

\(^1\text{For a few examples of recent contributions see Acemoglu and Johnson (2007), Shastry and Weil (2003), and Aghion and Durlauf, eds (2005)}\)
If we translate the concept of sector distribution to the number of major sectors in the economy, this finding can be rephrased in the following way. Along the path of development the number of major sectors follows an inverse U-shaped curve. The main goal of this paper is to examine under what economic conditions such dynamics might prevail.

The main claim of the paper is that such dynamics are a result of a very standard assumption in economics, namely the assumption of increasing marginal costs. More specifically, we claim that once sectors are set at increasing marginal costs, the number of sectors follows an inverse U-shaped curve. To see this, assume that there are two types of sectors in the economy, traditional and modern. The process of development shifts resources from traditional sectors to modern sectors, which are more productive. This defines a trade-off between traditional and modern sectors. The cost of setting an additional modern sector is therefore the number of traditional sectors that need to be eliminated in order to free resources for this new sector. This is the marginal cost of setting modern sectors and it is assumed to be increasing. As long as this marginal cost is smaller than one, development increases the number of total sectors in the economy. Once the marginal cost is higher than one, development reduces the number of sectors in the economy. This is our main heuristic explanation to the dynamics of diversification.

This heuristic argument must be carefully examined in a full fledged model of a growing economy. The reason for that is not purism, but awareness that the shift

\[\text{\textsuperscript{2}This result of non-monotonic diversification holds both within countries over time as well as across countries.}\]

\[\text{\textsuperscript{3}Acemoglu (2011) focuses theoretically on a different dimension of diversity, which is the diversity of technological progress and argues that a social planner would choose a more diverse research portfolio and would induce a higher growth rate than the equilibrium allocation.}\]
from traditional to modern sectors is itself generated by some deeper changes, like technical change or accumulation of human capital, which can themselves affect the sector composition of the economy. Hence, after presenting this basic idea, the paper turns into presenting a model of a growing economy, where sectors are determined endogenously.

The main mechanism in this model is the interplay between setup cost, skills distribution and productivity. Each sector in the model embodies some specific know-how. Each sector is comprised of starters, or innovators, and other workers. The starters develop the know-how, or bring it from abroad and adjust it to the local conditions. The workers supply their labor endowments. Here lies another major assumption of the model, namely that individuals are endowed with two types of abilities. One is raw labor, which is used only in the traditional sectors. The other ability is efficiency labor, and it is used only in the modern sectors. Individuals can choose which type of ability they sell in the market, raw labor or efficiency labor. Raw labor is assumed to be equal across individuals, while efficiency labor is randomly assigned across them. In equilibrium, individuals are endogenously split to unskilled, who use their raw labor only, either as starters or as producers in the traditional sectors, or to skilled who use their efficiency labor as starters or producers in the modern sectors.

Countries in this model differ by an exogenous parameter, which is the productivity of their modern sectors. Differences in this productivity can be a result of three possible reasons. One is development of technology over time. In this case the model can be interpreted as explaining the dynamics of diversification over time. Differences in productivity across countries can be a result of differences
in technology adoption across countries. There are a number of papers that discuss the reasons for potential differences in technology adoption across countries (Parente and Prescott 1994, Zeira 1998, Basu and Weil 1998). Moreover, there is quite significant empirical evidence that points at the possibility of such differences across countries (Caselli and Coleman 2006). Different productivity can be also a result of more standard reasons: differences in Geography, like climate, access to sea, natural resources or infrastructure, or institutions (McArthur and Sachs 2001, Acemoglu, Johnson and Robinson 2002, Sachs 2003). As productivity increases, more modern sectors are created, and the innovators and workers in these sectors come at the expense of the traditional sectors, whose number is reduced. As more people move to the modern sectors, their efficiency levels become lower and as a result more of them are needed to build each sector. Hence, the marginal reduction of traditional sectors becomes larger. In other words, this model displays increasing marginal costs to new modern sectors. It can therefore be shown that the total number of sectors follows an inverse-U shaped curve.

There have been few attempts to model theoretically the process of diversification. These models can be viewed as belonging to two separate strands, each predicts a monotonic relation between income and the change in diversification. Dornbusch, Fischer and Samuelson (1977) and Krugman (1991) predict a positive monotonic relation between income and sectoral concentration, whereas, Matsuyama (2000) and Acemoglu and Zilibotti (1997) predict a positive monotonic relation between income and sectoral diversification. We next briefly describe these theories.

Dornbusch et al. (1977) presents a Ricardian trade model with a continuum of
goods where the range of goods exported, non-traded, and imported is endogenously determined. In their model, a reduction in transportation costs (or tariffs) reduces the range of non-traded goods and thus enhances specialization. Krugman (1991) presents another type of argument that provides a different explanation for concentration. In Krugman (1991) the focus is the geographic agglomeration of economic activity. The key determinant of geographic agglomeration is the interaction of increasing returns, transportation costs, and demand. Naturally, this geographic agglomeration can be translated into sector concentration.

Models that predict a positive monotonic relation between income and diversification are mainly based on the structure of preferences. Non-homothetic preferences can explain sector diversification as economies develop, since agents with such preferences change the fraction of income spent on each good as income grows and that enables emergence of new sectors. Matsuyama (2000) develops a Ricardian model with non-homothetic preferences, which investigates the role of population size and technology on trade. The paper provides conditions under which diversification occurs as a result of exogenous increase in technology or in population. Acemoglu and Zilibotti (1997) introduce another explanation for diversification. In their model, diversification occurs endogenously as a result of agents’ decisions to invest in a range of imperfectly correlated risky projects, or “sectors”. However, not all risky projects are available at all points in time because of minimum size requirements. At an early stage of development countries do not invest in risky projects and thus do not diversify due to missing

\footnote{For recent survey see Neary (2001).}

\footnote{Recent studies acknowledge that the number and size of countries might also be endogenously determined. See Alesina and Spolaore (1997) and Alesina and Wacziarg (1998) among others.}
finance. As a result, economies accumulate capital gradually. Later in the development process, as the capital stock increases, and with it financial resources, countries diversify. Therefore: “development goes hand in hand with the expansion of markets and with better diversification opportunities” (p. 711). Clearly, our paper presents a different approach to diversification and our results differ significantly from these two types of literature. We share with these models the need to pay a fixed cost in order to build a sector, but we also point at a pool of limited resources, in our case human skill resources, which make the marginal costs of setting modern sectors increase with the process of development.

Our paper also examines some extensions of the basic model. First it shows that the main results hold not only for changes in productivity of the modern sector but also for changes in human capital. More specifically, we show that if the cost of education is reduced, so that more people acquire education, more modern sectors are created and less traditional sectors operate, and their number follows an inverse-U shaped curve as well. Hence, whether development is triggered by technical change or by improved public education, the effect on sector dynamics is the same. In addition the paper also examines what happens to the skill premium along the path of development and what happens to the gender wage premium as well.

The rest of the paper is organized as follows. In section 2 we present the main idea of the paper. Section 3 formalizes our argument in a model of skills. Section 4 presents a cross country analysis, while section 5 discusses some extensions of the model. Section 6 presents some concluding comments.
2 Main idea

Before turning to the detailed model, we wish to illustrate the main idea of the paper. We claim that the reason that the number of sectors is increasing and then decreasing during the process of development is a very basic assumption in economics, namely increasing marginal costs. In this case we refer to the costs of building new sectors. Consider an economy that produces in two types of sectors, traditional and modern, where the modern sectors are more productive than the traditional ones. Thus, the economy develops by closing down traditional sectors and by opening new modern sectors. Since the modern sectors are more productive than the traditional ones, this increases output and income. Assume that marginal costs are increasing in the following way: each new modern sector requires more resources and similarly each new traditional sector. As a result, each new modern sector replaces a larger number of traditional sectors. Denote by $J_T$ the number of traditional sectors and by $J_M$ the number of modern sectors. Due to increasing marginal costs the two numbers are related by a concave curve, which can be thought of as a production possibility frontier of sectors. We denote this curve by SPPF. The SPPF curve is shown in Figure 1.

Consider next the total number of sectors, traditional and modern. This number of sectors can be easily presented in Figure 1 as well. Consider the straight line with slope $-1$ that passes through the equilibrium point $E$, which lies on SPPF. This straight line intersects the horizontal axis, of $J_M$, at the number of sectors $J$. As the economy grows and $E$ moves along the SPPF to the right, namely as modern sectors replace traditional sectors, the number of sectors is increasing. But this rise in the number of sectors does not continue forever. As the economy
reaches $E^*$ the number of sectors reaches a maximum. After that it starts to decline as the economy keeps moving to the right along the SPPF. Hence, the number of sectors follows an inverse U-shape, just as found empirically by Imbs and Wacziarg (2003). Note that the point of maximum number of sectors $E^*$ is the point where the marginal cost of setting a new modern sector in terms of traditional sectors is equal exactly to 1.

Hence, if the marginal costs of setting a modern sector are increasing, the number of sectors in the economy increases, reaches a maximum, and then decreases during the process of economic development. But life is never that simple and we have to examine this basic idea by applying a more specific model. The reason is that growth is never limited to shifting the economy from traditional to modern sectors only. Usually the structure of the economy is a result of other changes,
driven by global technical progress or by human capital accumulation, or by both
and other changes. These changes also affect the number of sectors in the econ-
omy. Hence, in order to check whether our basic heuristic argument holds when
other things change we need to look at a more detailed and rich model. To that
we turn in the next section.

3 The model

Consider a small open economy that produces one final good, \( Y \), which is used
for consumption only. This final good is produced by two intermediate goods:
traditional good, \( T \), and modern good, \( M \). Each intermediate good is produced
by a discrete number of sectors. There are two factors of production in this econ-
omy: raw labor and efficiency units of labor. While traditional sectors produce
the traditional good using raw labor only, modern sectors produce the modern
good using efficiency units of labor only. All markets are assumed to be perfectly
competitive. The final good is assumed to be perfectly tradable, but labor as well
as the intermediate goods are assumed to be not tradable and their markets are
domestic. For simplicity there is no population growth and the population size
in each country is \( L \).

3.1 Individuals

Each individual is endowed with one unit of raw labor and also with a random
amount of efficiency units of labor. Individuals are uniformly distributed with
respect to efficiency units of labor over the segment \([0, \bar{h}]\). Individuals maximize utility from consuming the final good \(Y\).

### 3.2 Production of the Final Good

The final good is produced by the two intermediate goods according to the following CES production function:

\[
Y = (M^\rho + T^\rho)^{\frac{1}{\rho}} \quad \rho \in (0, 1),
\]

where \(M\) and \(T\) are the quantities of traditional and modern intermediates in the production of the final good, respectively.

### 3.3 Production of the Intermediate Goods

#### 3.3.1 Traditional Sectors

Setting up a traditional sector requires an exogenously given amount of \(l_T\) entrepreneurs. Each traditional sector produces the traditional good according to the following production function

\[
x_T = l^\alpha \quad \alpha \in (0, 1),
\]

where \(x_T\) is the traditional output, \(l\) is the input of raw labor workers employed in each traditional sector. Consequently, the profits that a traditional good producer
\(j\) maximizes are given by:

\[
\pi_{Tj} = P_T l_j^p - w_l l_j - w_l l_T, \tag{2}
\]

where \(\pi_{Tj}\) is the profit earned by the traditional sector \(j\) and \(w_l\) is the price of one unit of raw labor.

### 3.3.2 Modern Sectors

Setting up a modern sector requires a total amount \(h_M\) of efficiency units of labor. Each modern sector produces the modern good according to the following production function

\[
x_M = A h^\beta, \quad \beta \in (0, 1), \tag{3}
\]

where \(x_M\) is the modern output, \(h\) is the input of efficiency units of labor employed in each modern sector and \(A\) is a technological parameter, which is country specific. Consequently, the profits that a modern good producer \(k\) maximizes are given by:

\[
\pi_{Mk} = P_M A h_k^\beta - w_h h_k - w_h h_M, \tag{4}
\]

where \(\pi_{Mk}\) is the profit earned by the traditional sector \(k\) and \(w_h\) is the price of one efficiency unit of labor.
3.4 Factor Prices

3.4.1 The Final Good

The price of the final good is normalized to one. Profit maximization by producers of the final good $Y$, leads to the following first-order condition:

$$\frac{P_M}{P_T} = \left( \frac{T}{M} \right)^{1-\rho}, \quad (5)$$

where $P_M$ and $P_T$ are the prices of the modern and traditional intermediate goods, respectively. Given intermediate goods’ prices, cost minimization producers leads us to the usual ideal price index $P$, which we normalize to one and could be given by:

$$P = \left( P_{\rho M}^{\frac{\rho}{\rho - 1}} + P_{\rho T}^{\frac{\rho}{\rho - 1}} \right)^{\frac{\rho - 1}{\rho}} = 1. \quad (6)$$

3.4.2 The Traditional Good

Profit maximization by traditional sectors, which produce the traditional intermediate good, leads to the following first-order condition:

$$w_l = \frac{\alpha P_T}{l_j^{(1-\alpha)}}. \quad (7)$$

Since all traditional sectors face the same competitive equilibrium prices, it follows that for any traditional sector $j$

$$l_j = l. \quad (8)$$
Substituting equations (7) and (8) into (2) yields:

\[ \pi_T = (1 - \alpha) P_T l^\alpha - l_T w_l. \]  
(9)

The more traditional sectors emerge, the smaller are the rents of each one since each sector takes a smaller share of the market and, thus, employs less workers. Equilibrium takes place when each sector’s rent covers the set up cost. Thus, the zero profit condition and (7) and (8) imply:

\[ l = \frac{\alpha}{1-\alpha} l_T, \]
\[ w_l = \frac{\alpha P_T}{(1-\alpha)^{\frac{\alpha}{1-\alpha} l_T}}. \]
(10)

### 3.4.3 The Modern Good

Profit maximization by sector \( k \), which produces the modern intermediate good, leads to the following first-order condition:

\[ w_h = \frac{\beta P_M A}{h_k^{(1-\beta)}}. \]
(11)

Since all modern sectors face the same competitive equilibrium prices, it follows that for any modern sector \( k \):

\[ h_k = h. \]
(12)

Substituting (11) and (12) into (4) yields:

\[ \pi_M = (1 - \beta) P_M Ah^\beta - h_M w_h. \]
(13)
Again, equilibrium is reached when each sector’s rent covers its set up cost. Thus, at equilibrium, the zero profit condition yields:

\[(1 - \beta) P_M A h^\beta = h_M w_h, \]  

Substituting (11) and (12) in (14) yields

\[h = \frac{\beta}{1 - \beta} h_M,\]
\[w_h = \frac{\beta P_M A}{(\frac{\beta}{1 - \beta} h_M)^{(1 - \beta)}},\]  

### 3.5 Equilibrium

#### 3.5.1 Individuals

As all individuals are identical with respect to their raw labor they differ in their efficiency units endowments. Since traditional production requires raw labor only, the market efficiently allocate individuals with relatively low levels of efficiency units for the traditional domain while those with high levels of efficiency units for the modern domain. Consequently the marginal individual, who is endowed with \(h_0\) efficiency units of labor is indifferent between working in a traditional sector or a modern one. This implies that for such an individual

\[\frac{\alpha P_T}{(\frac{\alpha}{1 - \alpha} l_T)^{(1 - \alpha)}} = w_l = w_h h_0 = \frac{\beta P_M A}{(\frac{\beta}{1 - \beta} h_M)^{(1 - \beta)}} h_0.\]
3.5.2 Labor Market Clearing

Given (16), labor market clearing of raw labor implies

\[ J_T (l_T + l) = \int_0^{h_0} f(i) \, di, \]  

(17)

where \( J_T \) is the number of traditional sectors. Thus, the left hand side of (17) represents the demand for raw labor and the right hand side represents its supply.

Labor market clearing for the efficiency units of labor implies

\[ J_M (h_M + h) = \int_{h_0}^{\bar{h}} h_i f(i) \, di, \]  

(18)

where \( J_M \) is the number of modern sectors. Thus, the left hand side of (18) represents the demand for efficiency units of labor and the right hand side represents its supply.

3.5.3 Goods Markets Clearing

Equilibrium in the intermediate goods market occurs when

\[ J_T l^\alpha = T \]  

\[ J_M Ah^\beta = M \]  

(19)

are satisfied. The left hand side of (19) represents the supply for each intermediate good and the right hand side represents its demand.
3.6 Solution

While the equations in (10) solve for \( l \) and \( w_l \), the equations in (15) solve for \( h \) and \( w_h \), equation (16) solves for \( h_0 \), equations (17) and (18) solve for \( J_T \) and \( J_M \), the equations in (19) solve for \( T \) and \( M \), and equations (5) and (6) solve for \( P_T \) and \( P_M \).

Substituting \( h \) from (15) in (18), calculating the integral and isolating \( J_M \) yields

\[
J_M = \frac{1 - \beta}{\frac{h}{h_M}} \frac{L \hat{h}^2 - h_0^2}{2}.
\]  
(20)

Substituting \( l \) from (10) in (17), calculating the integral and isolating \( J_T \) yields

\[
J_T = \frac{1 - \alpha}{l_T} \frac{L}{h} h_0.
\]  
(21)

Isolating \( h_0 \) from (21), substituting it in (20) and rearranging yields

\[
\frac{J_M}{L} = \frac{(1 - \beta)}{2h_M} \left( 1 - \left( \frac{l_T}{1 - \alpha L} J_T \right)^2 \right).
\]  
(22)

**Proposition 1** The production frontier that governs the relation between the equilibrium number of modern sectors and traditional ones is concave.

**Proof:** It follows from equation (22) that \( \frac{\partial J_M}{\partial J_T} < 0 \) and \( \frac{\partial^2 J_M}{\partial J_T^2} < 0 \). Actually the relationship between the number of modern and traditional sectors is quadratic. \( \Box \)
4 Cross Country Analysis

As mentioned in section 2, growth is never limited to shifting the economy from traditional to modern sectors only. Usually the structure of the economy is a result of other changes, driven by global technical progress or by human capital accumulation, or by both and other changes. However, within our setting, countries vary with respect to many parameters, such as productivity, the pool of human capital, which is reflected by the distribution of efficiency units of labor, the cost of setting up a traditional or a modern sector and more. In our analysis below, we examine two key parameters that could pin down the differences in the structure of economies. These two key parameters are productivity and the pool of human capital.

4.1 Productivity

Remember that equation (22) summarizes a relationship between two endogenous variables: $J_T$ and $J_M$. To analyze the impact of a change in productivity, $A$ on the equilibrium structure we need to substitute for intermediate goods’ prices since productivity affects the composition of modern and traditional good in the production of the final good through prices.

Substituting the optimal employment of $l$ and $h$ from (10) and (15) into (19) and then substituting the outcome into (5) gives the relative price that is determined
by demand:

\[ \frac{P_M}{P_T} = \frac{1}{A^{1-\rho}} \left( \frac{J_T/L}{J_M/L} \right)^{1-\rho} \left( \frac{(\frac{\alpha}{1-\alpha})^{\alpha} l_T^{\alpha}}{(\frac{\beta}{1-\beta})^{\beta} h_M^{\beta}} \right)^{1-\rho}. \]  

(23)

Isolating the relative prices: \( P_M/P_T \) from (16) gives the relative price that is determined by supply:

\[ \frac{P_M}{P_T} = \gamma \left( \frac{h_M}{l_T} \right)^{1-\beta} \frac{1}{Ah_0}, \]  

(24)

where \( \gamma = \frac{\alpha^\rho(1-\alpha)^{1-\alpha}}{\beta^\rho(1-\beta)^{1-\beta}}. \)

Isolating \( h_0 \) from (21), substituting it into (24), substituting the result into (23) and rearranging gives:

\[ \delta \left( \frac{h_M}{l_T} \right)^{1-\beta} \frac{1}{h} = A^\rho \left( \frac{J_T/L}{J_M/L} \right)^{2-\rho}, \]  

(25)

where \( \delta = \frac{\alpha^\rho(1-\alpha)^{2-\alpha \rho}}{\beta^\rho(1-\beta)^{1-\beta \rho}}. \)

Proposition 2 describes the condition under which development, or substitution of traditional by modern sectors, is driven by a rise in productivity \( A \), the productivity of the modern sectors.

**Proposition 2** An increase in productivity increases the number of modern sectors on the expense of traditional ones along the concave frontier described in (22) if and only if \( \rho \in (0, 1) \). That is, the traditional good and the modern good are gross substitutes.

**Proof:** As (22) shows that \( (J_M/L) \) is decreasing with \( (J_T/L) \), (25) unambiguously determines that \( \frac{\partial J_T}{\partial A} < 0 \)

**Lemma 1** The level of threshold for efficiency units of labor decreases with development.
**Proof:** Follows directly from proposition (2)

### 4.1.1 Stages of Diversification

Proposition 2 states that if \( \rho \in (0, 1) \), then as \( A \) increases the number of modern sectors grows on the expense of the traditional ones. However, this result does not guarantee that the total number of sectors exhibits an inverted U-shaped pattern with respect to development, which we capture by the parameter \( A \). As depicted in figure 1, even with a concave production frontier as long as the slope is less than \(-1\), moving from traditional to modern sectors increases the total number of sectors and vice versa. Thus to guarantee the inverse U-shaped pattern of the total number of sectors a slope of \(-1\) must lie at an interior points on the frontier and this is what we examine next. Thus, the maximum number of sectors is located at the point where the slope of the production frontier, which is described by (22), is \((-1)\). at this point the number of traditional sectors, denoted by \( J_T^* \), is given by

$$\frac{J_T^*}{L} = \frac{(1 - \alpha)^2}{1 - \beta} \frac{h_M}{(l_T)^2},$$

(26)

and the number of modern sectors, denoted by \( J_M^* \) is given by

$$\frac{J_M^*}{L} = \frac{(1 - \beta)\bar{h}}{2h_M} \left( 1 - \frac{(1 - \alpha)^2}{(1 - \beta)^2} \frac{(h_M)^2}{(l_T)^2} \right),$$

(27)

Substituting (26) and (27) into (25) gives the level of productivity that yields the maximum number of sectors. This level of productivity, which is denoted by \( A^* \)
is given then by

\[ A^* = \frac{\lambda}{2} \left( \frac{1}{\rho} \right)^{2-(2-\alpha)\rho} \left( 1 - \frac{(1-\alpha)^2}{(1-\beta)^2} \right)^{\frac{1-\beta}{\rho}}, \tag{28} \]

where \( \lambda = \frac{\alpha^\rho (1-\beta)^{(2-2\rho)(1-\alpha)}}{\beta^\rho (1-\alpha)^{(2-2\rho)(1-\beta)}}. \) Notice that equation (28) shows that \( A^* \) is a finite number, which implies that for a sufficiently large range of productivity the cross sectional relationship between productivity and the number of sectors exhibits an inverted U-shape.

From (6) and (23) the prices of the traditional and modern intermediate goods at this point could be written as

\[ P_T^* = \left( 1 + \frac{1}{(A^*)^\rho} \left( \frac{J_T^*/L}{J_M^*/L} \right) \left( \frac{\alpha^{1-\alpha} l_T^p}{\beta^{1-\beta} h_M^\beta} \right) \right)^{\frac{1-\beta}{\rho}}; \tag{29} \]

and

\[ P_M^* = \left( \frac{1}{(A^*)^\rho} \left( \frac{J_T^*/L}{J_M^*/L} \right) \left( \frac{\alpha^{1-\alpha} l_T^p}{\beta^{1-\beta} h_M^\beta} \right) \right)^{\frac{1-\beta}{\rho}} + \left( \frac{1}{(A^*)^{2\rho}} \left( \frac{J_T^*/L}{J_M^*/L} \right)^2 \left( \frac{\alpha^{1-\alpha} l_T^p}{\beta^{1-\beta} h_M^\beta} \right)^2 \right)^{\frac{1-\beta}{\rho}}. \tag{30} \]

Finally, per-capita income is given by

\[ y^* = \frac{Y^*}{L} = \frac{J_T^*}{L} P_T^* \left( \frac{\alpha}{1-\alpha} \right)^\alpha + \frac{J_M^*}{L} P_M^* \left( \frac{\beta}{1-\beta} h_M \right)^\beta. \tag{31} \]

Therefore, from equations (26), (27), (28), (29) and (30) this per-capita income is
determined by the parameters of the model: \( \{\alpha, \beta, \rho, l_T, h_M, \bar{h}\} \).

### 4.2 Differences in Education Cost

Assume that to work in the modern sector individuals incur an education cost, \( x \) measured in terms of time. Clearly, this extension has no impact on the locus of the production frontier as this locus is affected by the distribution of raw labor, the distribution of efficiency units of labor and the parameters of the production of modern and traditional goods. However, this cost does affect the marginal individual and, therefore, determines the equilibrium point on the frontier. That is, the equilibrium number of modern and traditional sectors. Formally, equation (16) becomes:

\[
\begin{align*}
    w_l &= \frac{\alpha P_T}{(\frac{\alpha}{1-\alpha} l_T)^{(1-\alpha)}} = \frac{\beta P_M A(1 - x)}{(\frac{\beta}{1-\beta} h_M)^{(1-\beta)}} h_0 = w_h h_0 (1 - x). \\
    \text{(32)}
\end{align*}
\]

As evident from (32), a reduction in \( x \) affects the equilibrium number of sectors similar to an increase in \( A \). Intuitively, as an increase in productivity linearly increases output for a given level of efficiency units of labor so does a reduction of the cost of education \( x \) if it is given in terms of time. Thus, these two changes symmetrically affect the equilibrium number of sectors in the economy. Notice that a reduction of \( x \) reflects a provision of public schooling. Accordingly an increase in the provision of public education, which is an alternative source of growth could also explain the stages of diversification through the development process.
5 Effects on the Distribution of Wages

5.1 Skill Premium

Proposition 3 Skill premium, which is measured by the ratio of the income of the average individual working in the modern sectors relative to the one working in the traditional ones, increases with development.

Proof: Denote this ratio of income by $I$. Thus, $(I = w_h h_{av}/w_l)$, where $h_{av}$ is the average human capital of workers employed in the modern sectors: $h_{av} = (\bar{h} + h_0)/2$. from (16) we get that $w_h/w_l = 1/h_0$, which implies that $I = h_{av}/h_0 = \bar{h} / 2 h_0 + 1/2$. As $h_0$ decreases with development, inequality unambiguously increases. □

The intuition for this result is straightforward. It is always the case that the individual with $h_0$ efficiency units of labor is indifferent between working in a traditional sector or a modern sector since in both cases she earns the same wage. Since the number of modern sectors increases with development and so the range of individuals using their efficiency units of labor, the gap in terms of efficiency units of labor between the individual possessing the average efficiency units of labor, $h_{av}$ and that possessing the minimum efficiency units of labor, $h_0$ increases with development, which implies that the income gap between them increases as well.

5.2 Gender Differences

Could this framework account for differences between genders? To discuss this question assume that the population size is $2L$, which comprises $L$ males and
L females. We assume that raw labor is associated with physical intensive tasks and efficiency units of labor is associated with metal intensive tasks. It’s assumed further that males are stronger than females but with respect to mental endowment they are alike. Thus, while each male is endowed with one unit of raw labor and some efficiency units of labor, each female is endowed with $\gamma < 1$ units of raw labor and some efficiency units of labor. Males and females are uniformly distributed with respect to efficiency units of labor on the segment $[0, \tilde{h}]$.

These assumptions are supported by a number of empirical studies. Pitt, Rosenzweig and Hassan (2011) present evidence on the distribution of grip strength among adult males and females in the U.S. and rural Bangladesh. Their Appendix Figure 1 shows that in both populations men are substantially stronger than women, and that the distributions by gender are similar in both countries. Thomas and Strauss (1997) showed that these differences are relevant for labor market earnings. They found that in urban Brazil, body mass contributed to males’ earnings but not to females’.

Denoting the efficiency units of labor of the marginal male by $h_{0}^{ma}$ and the efficiency units of labor of the marginal female by $h_{0}^{fe}$, equation (16) then delivers the following threshold levels:

$$w_{l} = \frac{\alpha P_T}{(\frac{\alpha}{1-\alpha} l_T)^{(1-\alpha)}} = \frac{\beta P_M A}{(\frac{\beta}{1-\beta} h_M)^{(1-\beta)}} h_{0}^{ma} = w_{h} h_{0}^{ma},$$ (33)

for men and

$$\gamma w_{l} = \frac{\alpha P_T}{(\frac{\alpha}{1-\alpha} l_T)^{(1-\alpha)}} = \frac{\beta P_M A}{(\frac{\beta}{1-\beta} h_M)^{(1-\beta)}} h_{0}^{fe} = w_{h} h_{0}^{fe},$$ (34)

23
for women. From (33) and (34) we get that

$$\frac{h_{0}^{fe}}{h_{0}^{ma}} = \gamma.$$  \(\text{(35)}\)

### 5.2.1 Within versus between Gender Inequality

As a result of development, captured in our model by an increase in productivity, \(A\), the economy moves from traditional sectors to modern ones and, thus, both, \(h_{0}^{ma}\) and \(h_{0}^{fe}\) decline, which affects within gender as well as between gender inequalities.

**Proposition 4** As economies develop
(i) Within gender inequality increases.
(ii) Between gender inequality decreases.

**Proof:**
(i) See proof to proposition (3).
(ii) In the traditional sectors the gender gap is fixed at the level, \(1/\gamma\). However, in the modern sectors the ratio of the average income between the two sexes, which is denoted by \(R\) is

$$R = \frac{\bar{h} + h_{0}^{ma}}{\bar{h} + h_{0}^{fe}}.$$  \(\text{(36)}\)

Substituting (35) in (36) yields

$$R = \frac{\bar{h} + \gamma h_{0}^{ma}}{\bar{h} + \gamma h_{0}^{ma}}.$$  \(\text{(37)}\)

Equation (37) reveals that \(\frac{\partial R}{\partial h_{0}^{ma}} > 0\). As lemma (1) states that \(\frac{\partial h_{0}^{ma}}{\partial A} < 0\) it turns out that \(\frac{\partial R}{\partial A} < 0\).
A word of intuition is useful here. First, with regard to the within gender inequality, our model assumes that the variation across individuals with respect to efficiency units is greater that of raw labor. As a result, a migration from the traditional sectors to the modern ones increases within gender inequality. This is true for male workers as well as female workers. Second, with regard to the between gender inequality, our model abstracts from unemployment issue so all males and females are fully employed by assumption. Since females have relative advantage in producing the modern good as they are endowed with less units of raw labor, $\gamma < 1$, the ratio of females to males in the modern sector is more than one. Thus, the average female has less efficiency units of labor than the average male has and a gender wage gap exists. As productivity increases, the constant productivity in the traditional sectors implies that more males join the modern sectors than females do, which shrinks the differences in the average skills between the genders and so does the difference in wages.\footnote{for empirical support for our explanation see Welch (2000) for U.S. data and Gosling (2003) for British data.}

6 Conclusion Remarks

This paper presents a theory of how sectors are endogenously created in an economy and what affects their number or distribution. The main result of the paper is that, as the economy develops, initially sectors become less concentrated, namely economic activity is spread across a larger variety of sectors but there exists a level of development beyond which sectors begin to concentrate again across smaller variety of sectors. In other words, the number of sectors follows
an inverse U shaped curve consistent with the evidence provided by Imbs and Wacziarg (2003).

The paper distinguish between modern and traditional sectors and builds on a very standard assumption in economics, which is the assumption of increasing marginal costs. Assuming that individuals are equally endowed with raw labor, while efficiency labor is randomly assigned across them gives rise to increasing marginal cost of setting up new sectors. Since throughout the development process economies move from traditional sectors to modern ones and since individuals with more efficiency units are employed first, the marginal reduction of traditional sectors increases with the number of modern sectors.

References


