Economic Growth and Sector Dynamics*

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Abstract

This paper analyzes the endogenous determination of sectors in a growing economy. It assumes that there are traditional sectors and modern sectors and economic growth is driven by rising productivity of the modern sectors. It also assumes that individuals are heterogeneous, which leads to increasing marginal opportunity costs in creating new modern sectors. We show that under these two main assumptions, economic growth first increases diversification to sectors and then reduces it. We also show that for the equilibrium to be stable and well-behaved, it is required that the modern and traditional sectors should be substitutes and not complements.

Keywords: Traditional sectors, Modern sectors, Diversification, Economic Growth, Substitutability.

JEL Classifications: L11, L16, L22, O11, O41.

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1. Introduction

Recent decades have seen a dramatic increase in research on economic growth and development.[1] Naturally, this literature is mainly macroeconomic and it focuses on how aggregate output rises over time and why it differs across countries. But economists have always known that the process of economic growth is not only aggregate, but entails deep structural changes over time. The obvious and most famous one is the move from agriculture to industry and later from industry to services.[2] But the structural changes that come with development are much wider than this change alone. Economic growth changes continuously the set of goods produced and the set of goods consumed in a country. Therefore, growth leads to continuing changes in the structure of sectors. Unfortunately, this process has not received sufficient attention in the wide research on economic growth.[3]

This paper contributes to this area of research by introducing a model that connects the dynamics of economic growth to the dynamics of the sector structure in the economy. The model has two types of sectors, traditional and modern, and economic growth is driven by increasing productivity in the modern sectors. As a result economic growth leads to transition of production from the traditional to the modern sectors, where the number of traditional sectors declines and the number of modern sectors increases. The model reaches two main results. The first is that sector diversification in the economy follows an inverse U shaped curve along economic growth, first rising and then declining. This result is interesting and important because it fits well the empirical findings of Imbs and Wasciarg (2003). The second main result of the model is that the elasticity of substitution between the modern and traditional goods should be greater than one for the equilibrium to be well behaved. We also show that due to similar reasons the elasticity of substitution between the modern sectors themselves and between the traditional sectors themselves should be even higher.

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[1] This broad area of research is surveyed in two volumes of the Handbook of Economic Growth, Aghion and Durlauf (2005, 2014).

[2] See early work by Kuznets (1966).

[3] Only one chapter in the two volumes of the Handbook of Economic Growth, Herrendorf, Rogerson and Valentinyi (2014), discusses sectors. It focuses on the three main sectors agriculture, industry and services.

The model in this paper builds on a number of assumptions, some are crucial for the main results and some are only made for simplification. One main assumption is that there are two types of sectors, traditional and modern. The second main assumption is that each type of sectors employs a different type of labor, raw labor in traditional sectors and efficiency labor in modern sectors. The third main assumption is that efficiency labor, which is used in modern sectors, differs across people. The fourth main assumption is that productivity of modern sectors is rising exogenously over time and this is the engine of growth in the model. We treat technical change as exogenous, as we view countries as small open economies that use mainly technologies that are invented elsewhere.[4] The fifth assumption of the model is there is some size requirement to production, in order to avoid the case of infinite number of sectors. We consider two types of such requirements, one at the firm level, in the benchmark model, and one at the sector level in an extension in Sub-Section 6.1. While the two assumptions imply different market structures, monopolistic competition and perfect competition respectively, they do not affect the main two results of the model. Another assumption used in the model is that the products of the various sectors are intermediate inputs in the production of the aggregate good, and are not consumed directly. This assumption as well does not have significant effects on the main results of the model.

Sector dynamics in this model can therefore be described as follows. As productivity in modern sectors increases, income in these sectors rises, so more workers choose to supply efficiency labor instead of raw labor and move to modern sectors. New modern firms are created and instead of locating in existing sectors, where higher output lowers the price, firms prefer to open new modern sectors, where prices are higher.[5] Hence, the number of modern sectors increases and the number of traditional sectors declines. As more people supply efficiency labor, its average efficiency declines, since efficiency labor is supplied by people with the highest efficiency. Hence, each new modern sector requires resources from more traditional sectors. As a result the marginal opportunity cost, in terms of traditional sectors, of building a new modern sector is increasing. This affects the total number of sectors. As a new modern sector is set, the total number of sectors increases by one minus the marginal opportunity cost of this sector. As long as this marginal cost is lower than one, the total

[4] In Section 2 we elaborate more on this assumption and supply evidence to support it.

[5] This process resembles the literature on diversification to states, as it is described in Alesina and Spolaore (1997) and Alesina and Wasciarg (1998).

number of sectors increases with economic growth. Once the marginal cost exceeds one, the total number of sectors begins to decrease. Hence, diversification of sectors increases up to some level of development and from there on it declines. This result, of an inverted U path of sector diversification along economic growth, is an important result of the paper.

As mentioned above, this result fits well the empirical study of Imbs and Wasciarg (2003), who find that diversification to sectors first increases with economic growth up to some level of per capita income, but beyond it sector diversification tends to decline. This result holds both over time and across countries. Our explanation to this inverse U path of diversification relies on a very basic assumption in economics, namely increasing marginal costs. The second main result of the paper is on substitutability between sectors. In the solution of equilibrium of the model, we realize that to have a stable and well-behaved equilibrium we need to add some restrictions on the main parameters of production. These restrictions imply that the elasticity of substitution between modern and traditional goods should be higher than one and also that the elasticity of substitution between the modern intermediate goods themselves and between the traditional intermediate goods themselves should be even higher than the substitution between the modern and the traditional goods. Note that modern and traditional productions require different types of labor, which can be also correlated with education. Interestingly, empirical studies, summarized in Caselli and Coleman (2006), have shown that the elasticity of substitution between high-educated and low-educated labor is around 1.4. Since the elasticity of substitution between the relevant goods should be close to it, we can view it as some supporting evidence.

There are two main literatures that are closely related to our paper. The first literature is on sector diversification, and the second literature is on diverging growth paths across sectors. The first literature, on diversification, can be divided to two separate strands, where each highlights a different relation between growth and diversification. Dornbusch, Fischer and Samuelson (1977) predict a negative monotonic relation between development and diversification, since reduction in transport costs reduces the set of non-traded goods and leads to concentration. Krugman (1991) also predicts a negative relation between growth and diversification, but the mechanism he suggests is geographic agglomeration. Another part of this literature reaches an opposite prediction, namely that income and diversification should be positively related. Matsuyama (2000) assumes non-homothetic preferences, so when income rises, agents spend less on each good and new sectors emerge. Acemoglu and Zilibotti (1997) view sectors as risky projects, which have a minimum size. Hence, higher income enables a greater diversification of risk, which implies more sectors. Regev and Zoabi (2014) also predict a positive relation between income and diversification, but their mechanism builds on talent utilization. This paper differs from the rest of this literature as it reaches a result of non-monotonic relation between growth and diversification.

Another recent literature studies unbalanced growth of sectors. It mainly tries to explain the dynamics of the shares of the three main sectors, agriculture, industry and services. This paper is related to this literature, since it also has two types of sectors, traditional and modern, which grow at different rates. This literature can also be divided to two opposing lines of research. In the first the sector with faster technical change attracts more labor and becomes dominant. Echevarria (1997), Laitner (2000), Kongsamut, Rebelo and Xie (2001), Gollin, Parente and Rogerson (2002) and Foellmi and Zweimuller (2008) are examples of such a result. In the second line of research the low growing sectors attract most labor and dominate the economy. Examples of such results are Baumol (1967), Caselli and Coleman (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2007). Due to our result of high substitutability of the two main goods, this paper is closer to the first line of research. The reason for that is due to the types of sectors and types of inputs used in them, as explained in the paper at the discussion of the result of substitutability.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the main equilibrium conditions. Section 4 derives the general equilibrium in the economy, and examines the implications of having a well-behaved and stable equilibrium for the substitutability between the two types of goods. Section 5 discusses the relation between growth and diversification. Section 6 analyzes two extensions of the model. Section 7 concludes. The Appendix contains proofs.

2. The Model

Consider an economy that produces one final good Y, which is used for consumption only. The final good is produced by two aggregate goods: a traditional good, T, and a modern good, M. Each of the two aggregate goods is produced by a discrete number of intermediate goods, each produced in a separate sector. The numbers of the intermediate goods, J_T and J_M, which are also the numbers of sectors of each type, are determined endogenously. There are two main factors of production: raw labor and efficiency labor. While traditional sectors use raw labor, modern sectors use efficiency labor. The markets for labor and for the aggregate goods, which are the final, the traditional and the modern goods, are perfectly competitive. The economy is open and small. While the final good is tradable, labor as well as intermediate goods are non-tradable and their markets are domestic.

2.1 Individuals

Individuals live one period each in non-overlapping-generations. Assume that there is no population growth and the population size in each generation is L. Each individual is endowed with one unit of raw labor and is also endowed with a random amount of efficiency labor. This amount of efficiency labor, denoted e, is uniformly distributed across people over $[0, E]$. A person chooses whether to supply raw labor or efficiency labor but she cannot supply both. When making this choice people maximize utility, which is from consumption of the final good is: $u = c$. Hence, they choose the type of labor that maximizes income. In each sector labor can be hired for production or for expertise, where experts acquire 'know-how' of their respective sector. We assume that this acquisition is costless, so that workers and experts are perfectly substitutable and earn the same income.[6] This holds both for raw labor in traditional sectors and for efficiency labor in modern sectors. For the sake of simplicity we assume that expertise is independent of the random level of efficiency labor.

2.2 Production of the Final Good, the Traditional Good and the Modern Good

The final good is produced from the traditional and the modern aggregate goods according to the following CES production function:

$$(1) \qquad Y = \left(Y_T^{\rho} + Y_M^{\rho}\right)^{\frac{1}{\rho}}.$$

Y is the quantity produced of the final good, while Y_T and Y_M are the quantities of the traditional and the modern goods, respectively. The traditional good itself is produced by intermediate goods from the traditional sectors according to:

$$(2) \qquad Y_T = \left(\sum_{j=1}^{J_T} y_{T,j}^{\sigma}\right)^{\frac{1}{\sigma}}.$$

[6] In Sub-Section 6.1 we examine the case of costly acquisition of 'know-how.'

In this CES production function $y_{T,j}$ is the quantity of intermediate good j produced in traditional sector j. Similarly, the aggregate modern good is produced by the following CES production function, where $y_{M,j}$ is the quantity of intermediate good j produced in modern sector j:

(3) $$Y_M = \left(\sum_{j=1}^{J_M} y_{M,j}^{\sigma} \right)^{\frac{1}{\sigma}}.$$

The parameters of the CES production functions ρ and σ are generally between -∞ and 1. We do not impose here any further restrictions on these parameters, except for assuming that σ is identical for traditional and modern production. As we analyze the equilibrium below we realize that some additional restrictions on these parameters are required to ensure a stable and well-behaved equilibrium.

2.3. Production in Sectors

Each of the traditional and modern intermediate goods is produced in a separate sector by production workers and by experts. Each firm i in a traditional sector j requires expertise at a quantity x of raw labor at any level of production. Its output depends on the amount of raw labor in production $l_{j,i}$ as follows:

(4) $$y_{T,j,i} = l_{j,i}^{\alpha}.$$

The parameter α is between 0 and 1. Each firm i in a modern sector j also requires expertise at a quantity x of efficiency units of labor for any level of production. Its output depends on the amount of efficiency labor in production $h_{j,i}$ in the following way:

(5) $$y_{M,j,i} = Ah_{j,i}^{\alpha}.$$

The parameter A is the productivity of modern sectors. A can be also interpreted as the state of technology in the country, which is common to all modern sectors and is further discussed in the next sub-section.[7]

[7] We assume for simplicity that traditional and modern sectors are quite symmetric, except for productivity. These assumptions are made for simplification only and do not affect the results at all.

2.4. Technical Change

The model's main engine of growth is technical change, which applies to modern sectors only. We further assume that technology A is exogenous for the country. These two assumptions require some explanation. The assumption that growth is mainly a result of technical change is fairly realistic. Usually economists consider both technical change and acquisition of human capital to be the two main engines of growth. But historical evidence shows that accumulation of human capital accounts for only a small part of economic growth in the last two centuries.[8] The assumption that technical change is exogenous can be justified by two arguments. First, since most R&D is done by a small group of countries, especially the US, we can assume that there is a global technological frontier, which increases over time, and countries just follow it, treating it as exogenous. But some countries do not follow this global frontier fully, as is well documented in the literature.[9] In this paper we assume that this partial adoption of technology is exogenous as well. This is a simplifying assumption, but it is also based on some supporting evidence. A recent empirical study by Battisti, Di Vaio and Zeira (2014) finds that the degree of following the frontier across countries depends on variables like climate, ethnic diversity or openness to trade, but not on the sector structure of the economy. Thus, in a model that focuses on sector dynamics, we can assume that technical change in the country is exogenous.

3. Equilibrium Conditions

3.1 Profit Maximization of Aggregate Goods

The price of the final good is normalized to one. The prices of the traditional and the modern goods are denoted P_T and P_M, respectively. The prices of the intermediate goods, traditional and modern, are denoted $p_{T,j}$ and $p_{M,j}$, respectively. The wage of raw labor is denoted w_l, and this is also the wage of experts, since we assume that acquisition of 'know-how' is costless.

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[8] According to Maddison (2005) GDP per capita in the developed countries increased 20 times since 1820. Average schooling increased by 10 years since then, so by standard development accounting calculations, human capital increased only 3 times.

[9] For empirical evidence on differences in technology across countries see Acemoglu and Zilibotti (2001), Caselli (2004), Caselli and Coleman (2006) and Comin and Hobijn (2010). For explanations for such differences see Parente and Prescott (1994), Zeira (1998), Basu and Weil (1998), and Brunt and García-Peñalosa (2011).

The wage of one unit of efficiency labor is denoted w_h, and this is also the wage of an efficiency unit hired as expertise in a modern sector. Profit maximization by producers of the final good leads to the following first-order conditions:

$$P_T = Y^{1-\rho} Y_T^{\rho-1},$$

and:

$$P_M = Y^{1-\rho} Y_M^{\rho-1}.$$

We next turn to profit maximization by producers of the traditional and the modern goods. The profits from production of the traditional good are:

$$P_T \left(\sum_{j=1}^{J_T} y_{T,j}^{\sigma} \right)^{\frac{1}{\sigma}} - \sum_{j=1}^{J_T} p_{T,j} y_{T,j}.$$

Hence, the first order condition with respect to input of intermediate traditional good j is:

(6) $$p_{T,j} = P_T Y_T^{1-\sigma} y_{T,j}^{\sigma-1} = Y^{1-\rho} Y_T^{\rho-\sigma} y_{T,j}^{\sigma-1}.$$

This first order condition describes the inverse demand for the traditional intermediate good j.

Equation (6) implies that the price is falling with the quantity of $y_{T,j}$ purchased, since $\sigma - 1 < 0$. That creates an incentive for firms to prefer to create a new sector, rather than enter a sector that already exists, where there is at least one firm, since the price of the intermediate good will be higher in the new sector.[10] As a result, there is only one firm in each sector in equilibrium and it is therefore a monopolist. This story holds on one condition only, if there is demand for a new intermediate good from a new sector. To examine this condition we calculate the derivative of the above profits with respect to the number of goods J_T and get:

$$P_T \frac{1}{\sigma} Y_T^{1-\sigma} y_{T,J_T}^{\sigma} - p_{T,J_T} y_{T,J_T} = \left(\frac{1}{\sigma} - 1 \right) P_T Y_T^{1-\sigma} y_{T,J_T}^{\sigma}.$$

[10] We assume implicitly that the numbers of sectors, J_T and J_M, are high, so the effect of opening a new sector on Y_T is negligible. This is also the reason why we treat the variables J_T and J_M as real numbers in the following analysis.

This implies that if σ is negative, producers of the traditional good T do not buy an additional intermediate good at the market price, since it reduces their profits. As a result, if $\sigma < 0$ the number of traditional sectors in equilibrium will be 1. Hence, in this case a theory of sectors becomes redundant. The same problem arises with respect to modern sectors. To avoid such an outcome, we therefore add the following restriction:

Restriction 1: The parameter σ is positive: $\sigma > 0$.

Hence, every traditional sector has only one firm, which is a monopoly. Since Restriction 1 holds also for the modern sectors, this conclusion holds for the modern sectors as well.[11] In these sectors the first order condition is derived similarly and it is:

(7) $$p_{M,j} = P_M Y_M^{1-\sigma} y_{M,j}^{\sigma-1} = Y^{1-\rho} Y_M^{\rho-\sigma} y_{M,j}^{\sigma-1}.$$

The result that in both types of sectors each sector contains only one firm, which is a monopoly, depends not only on Restriction 1, but also on our assumption that 'know-how' acquisition is costless. If such acquisition is costly, these costs are mitigated by flows of 'know-how' between firms in a sector, which give rise to increasing returns to scale. Sub-Section 6.1 shows that when this happens, there can be many firms and as a result competition in each sector.

3.2 Profit Maximization in Individual Sectors

Profits of the only traditional firm in traditional sector j, which is a monopolist, are:

$$\pi_{T,j} = p_{T,j} y_{T,j} - w_l l_j - w_l x.$$

Applying the demand price of this intermediate good from equation (6) and the production function (4) we get that profits are equal to:

(8) $$\pi_{T,j} = Y^{1-\rho} Y_T^{\rho-\sigma} y_{T,j}^{\sigma} - w_l l_j - w_l x = Y^{1-\rho} Y_T^{\rho-\sigma} l_j^{\alpha\sigma} - w_l l_j - w_l x.$$

Maximization of monopolistic profits yields the following first order condition:

(9) $$\alpha\sigma Y^{1-\rho} Y_T^{\rho-\sigma} l_j^{\alpha\sigma-1} = w_l.$$

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[11] This result relates our paper to Murphy, Shleifer and Vishny (1989), which also has sectors with monopolistic competition, but apart from that the papers are very different.

Substituting this first order condition in (8), we get that the maximized profits in a traditional sector are:

(10) $$\pi_{T,j} = w_l \left(\frac{1-\alpha\sigma}{\alpha\sigma} l_j - x \right).$$

Similar profit maximization in modern sectors yields the following first order condition that describes the wage of efficiency labor:

(11) $$\alpha\sigma Y^{1-\rho} Y_M^{\rho-\sigma} A^{\sigma} h_j^{\alpha\sigma-1} = w_h.$$

A similar calculation of profits shows that the maximized profits in a modern sector are:

(12) $$\pi_{M,j} = w_h \left(\frac{1-\alpha\sigma}{\alpha\sigma} h_j - x \right).$$

3.3. Zero Profit Conditions

As long as profits of each sector are positive, there are incentives to enter and create new firms, namely new sectors. This entry continues until profits are equal to zero. From the profit equations (10) and (12) it follows that the zero profit conditions determine the amounts of production labor in traditional and in modern sectors, which happen to be the same:

(13) $$l_j = h_j = \frac{x\alpha\sigma}{1-\alpha\sigma}.$$

This equilibrium condition enables us to understand the role of the requirement of x experts in each firm. It sets a size for the firm and thus for the sector. This size is also the reason why we end up with a finite number of sectors, since otherwise the assumption that $\sigma > 0$ might lead to infinite number of sectors.

Equation (13) also implies that all traditional sectors hire the same amount of labor l and thus each sector produces the same amount of output:

$$y_{T,j} = y_T = \left(\frac{x\alpha\sigma}{1-\alpha\sigma} \right)^{\alpha}.$$

Similarly all modern sectors hire the same amount of efficiency labor h and each sector produces the same amount of output:

$$y_{M,j} = y_M = A\left(\frac{x\alpha\sigma}{1-\alpha\sigma}\right)^{\alpha}.$$

3.4 Labor Markets' Equilibrium Conditions

Individuals are identical with respect to their raw labor but differ in their efficiency labor endowments. Since their income in the modern sectors is proportional to this endowment, people with relatively low levels of efficiency labor go to work in the traditional sectors, either as producers or as experts, while people with high levels of efficiency labor go to work in modern sectors, as producers or as experts. We denote by *s* the threshold level of efficiency above which people work in modern sectors and below it in traditional ones. At *s* an individual is indifferent between working in the two types of sectors, hence the following condition holds:

(14) $$w_l = w_h s.$$

We next turn to market clearing conditions of the two types of labor. We derive in equation (13) the number of workers in each traditional sector and the amount of efficiency labor in each modern sector. To that we should add the amount of expertise in each sector. Together we get the total amount of employment in each sector, which we denote by *m*:

$$m = x + \frac{x\alpha\sigma}{1-\alpha\sigma}.$$

Multiplying employment by the numbers of traditional and modern sectors yields the demands for raw labor and for efficiency labor respectively. The supplies of the two types of labor are determined by the threshold efficiency level *s* between the two types of labor and by our assumption on the uniform distribution of efficiency labor.[12] Hence, the labor market clearing condition for raw labor is:

(15) $$J_T m = \int_0^s \frac{L}{E} de = \frac{L}{E} s.$$

Similarly, the market clearing condition for efficiency labor is:

[12] The results would be similar if the distribution of efficiency would be different.

(16) $$J_M m = \int_s^E e \frac{L}{E} de = \frac{L}{E} \frac{E^2 - s^2}{2}.$$

4. General Equilibrium

In this section we put together the equilibrium conditions and show how they determine general equilibrium in the economy. This is done in a few steps. We first use the equilibrium conditions to describe the joint determination of two variables, the threshold between the two types of labor *s*, and the relative size of the two types of sectors, the ratio between the number of traditional sectors and the number of modern sectors, J_T / J_M. We show that in order to have out-of-equilibrium stability in each period and for the equilibrium to be well-behaved in very intuitive ways, we need to add some more restrictions on the parameters ρ and σ. We then show how the numbers of each type of sectors, J_T and J_M, are determined. The rest of the general equilibrium, including prices, wages and quantities, is fully determined by the equilibrium conditions, which are described in Section 3 above.

4.1 Determination of the Threshold between Raw and Efficiency Labor

We begin the derivation of general equilibrium with dividing the labor markets clearing conditions, equations (15) and (16), by one another. We get the following condition that reflects equilibrium in the two labor markets:

(17) $$\frac{J_T}{J_M} = 2 \frac{s}{E^2 - s^2}.$$

This positive relationship is described by the curve G in Figure 1 below, which is upward sloping but bounded by *E*.

Another relationship between these two variables is derived from equation (14), according to which the threshold efficiency level *s* should be equal in equilibrium to the wage ratio between the two types of labor, due to mobility between them. This ratio between the wage of raw labor and the wage of efficiency labor can be derived from the first order conditions, (9) and (11), and we get after some manipulation:

(18) $$s = \frac{w_l}{w_h} = A^{-\sigma}\left(\frac{Y_T}{Y_M}\right)^{\rho-\sigma} = \left(\frac{J_T}{J_M}\right)^{\frac{\rho-\sigma}{\sigma}} A^{-\rho}.$$

To simplify notation denote from here on $\chi = (\rho - \sigma)/\sigma$. Equation (18) is drawn in Figure 1 as curve M, since it describes labor mobility between the two types of sectors. The specific curve M in Figure 1 is drawn under the assumption that χ is negative. Equilibrium of the economy requires that both (17) and (18) hold, namely that the two curves, G and M, intersect. We denote this equilibrium point in Figure 1 by Q and it determines the equilibrium threshold between raw and efficiency labor *s* and the ratio between the sizes of the two aggregate sectors, J_T / J_M, which we denote by *z*.

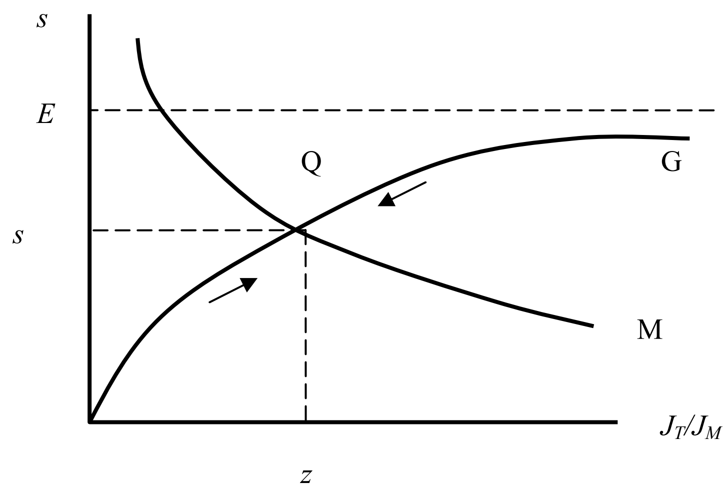

Figure 1

We first examine the existence of equilibrium. If χ is negative, as in Figure 1, the equilibrium always exists and is unique, since the M curve goes from infinity at left to zero at right. If $\chi = 0$, the M curve is horizontal and the equilibrium exists if $A > E^{-1/\rho}$ and it is unique. If productivity is lower in this case, equilibrium is reached at $s = E$ and there are no modern sectors. We next discuss out-of-equilibrium stability of the unique equilibrium. Note that the curve M informs us on how the economy behaves out-of-equilibrium. If the economy is above M, namely if $s > w_l / w_h$, the marginal individual earns more from efficiency labor than from raw labor, so people move from raw to efficiency labor, and *s* is reduced.

Similarly, below M, the threshold s is increasing. Hence, only an equilibrium point, in which M intersects G from above, is stable. Clearly if χ is negative, as assumed in Figure 1, the unique equilibrium point is stable and this holds also for the case of $\chi = 0$. The following subsection explains why we restrict the model to non-positive χ, and it also adds a restriction on the parameter ρ.

4.2 Further Restrictions on the Parameters and Substitutability

Appendix 1 shows that if χ is positive the equilibrium is either unstable, or it leads to production only in traditional sectors for any level of productivity A, or it is not continuous with changes in A. All these three outcomes are problematic and a theoretical model of sector dynamics should avoid them. First, we would like to have a stable equilibrium. Second, it is not reasonable to have equilibrium with only traditional production, if the productivity of modern sectors is very high. Third, the discontinuity of equilibrium with respect to A means that the economy produces only in traditional sectors if A is low, but at some level of A it jumps to production with a positive number of modern sectors. Such discontinuity should also be avoided in a theoretical model. As a result of these considerations we restrict the model from here on in the following way:

Restriction 2: The parameter χ is non-positive, namely: $\rho \leq \sigma$.

Note that changes in A have no effect on the G curve, but they shift the M curve in a way that depends crucially on the sign of the parameter ρ. Assume first that ρ is negative. In this case, if productivity of modern sectors A is reduced to zero, the M curve shifts down and the equilibrium s falls to zero. This means that more and more workers move to the modern sectors as they become less and less productive. In the limit, when their productivity is zero, the economy is producing only modern goods, namely producing zero. This result is highly implausible since the economy converges to a state with zero output, while it can produce much more using raw labor in traditional sectors. A similar problem arises if ρ is zero. In this case changes in A have no effect on the equilibrium s, which remains lower than E even if A is very low. This is again an implausible result, where production in modern sectors is unchanged, even if their productivity falls to zero. Hence, both the cases of negative and zero ρ lead to implausible results and that brings us to the following restriction:

Restriction 3: The parameter ρ is positive, $\rho > 0$.

Hence, our analysis of equilibrium leads us to impose some restrictions on the parameters of the CES production functions in the model in order to have a well behaved and stable equilibrium. These required restrictions on the parameters are therefore: $\sigma > 0$, $\rho > 0$, and $(\rho - \sigma)/\sigma \leq 0$. We can summarize these three restrictions in the following way:

(19) $$\sigma \geq \rho > 0.$$

Since both ρ and σ are from CES production functions, they are related to the elasticities of substitution. Thus, equation (19) implies the following result of the paper:

Proposition 1: The three restrictions on the parameters of the CES production functions, which are required in order to have a stable and well-behaved equilibrium, imply that the elasticity of substitution between modern and traditional goods is higher than 1. They also imply that the elasticity of substitution between the traditional intermediate goods themselves and between intermediate modern goods themselves is even higher than the elasticity of substitution between traditional and modern goods.

As discussed in the introduction, the literature is divided on this issue of substitution between the two types of goods, which is related to the issue of which of the two goods attracts more inputs, the one with high productivity growth or the one with low productivity growth. As discussed in the introduction this paper belongs to the first line of research. Our conclusion of high substitutability puts this paper in contrast especially with Acemoglu and Guerrieri (2007), who reach an opposite conclusion on substitutability. We next try to explain the reason for this difference. Our model considers modern sectors and traditional sectors, which can be thought of as two types of methods of production that can substitute one another. In Acemoglu and Guerrieri (2007) the two types of sectors are very different, as one sector is capital intensive and the other is labor intensive. Since capital and labor are complementary, it follows that the two goods, the capital intensive and the labor intensive, will be complementary as well. In our model the two inputs are raw labor and efficiency labor, which are not complementary at all. Hence these differences between the models lead to different conclusions on substitutability. Interestingly, raw labor and efficiency labor of our model are close in spirit to low-educated and high-educated labor. According to Caselli and Coleman (2006) the empirical literature on the elasticity of substitution between skilled and unskilled labor finds that it is between 1 and 2, where a more precise estimate is 1.4. These numbers fit the results of Proposition 1 well.

4.3 Determination of the Numbers of Traditional and Modern Sectors

Figure 1 describes how the equilibrium ratio between the sizes of the two types of sectors z is determined. We next turn to find out how each of these sizes, namely J_T and J_M, are determined. From the labor market clearing conditions in the two markets, equations (17) and (18), we derive the following relation between the two sizes:

$$\frac{J_T}{L} = \frac{1}{m}\sqrt{1 - \frac{2m}{E}\frac{J_M}{L}}. \tag{20}$$

Note that the square root is always well-defined, since $J_M m$ is the total efficiency labor hired, while $LE/2$ is the total efficiency labor in the economy, so the ratio between them must be smaller than 1. Equation (20) describes a negative and concave relation between the number of modern sectors J_M and the number of traditional sectors J_T. This relation, which we call the Sector Frontier, is described by the curve F in Figure 2. It fits our intuitive description of the increasing marginal opportunity costs of creating new modern sectors in the Introduction. In our model these increasing marginal costs are a result of the heterogeneity of efficiency labor. As more modern sectors are created, more efficiency labor is hired and as a result its average level is reduced, as the higher-efficiency workers are hired first. This increases the cost of creating new modern sectors. Note that the Sector Frontier does not depend at all on the productivity of modern sectors A.

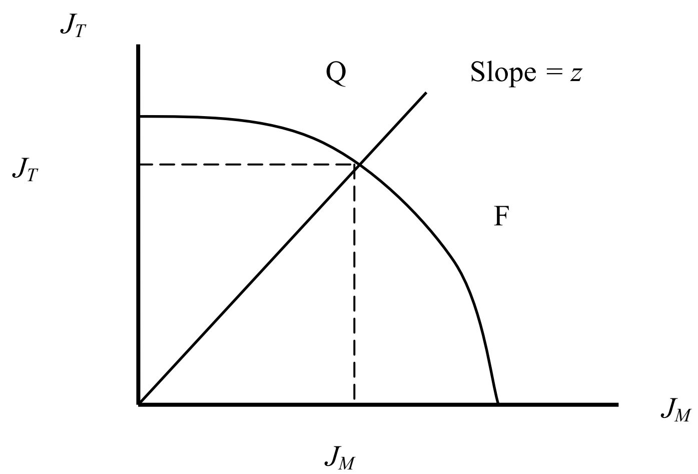

Figure 2

Figure 2 describes the determination of the equilibrium numbers of traditional and modern sectors in the economy. The curve F describes the relationship between these two variables, while the diagonal line at slope z describes the equilibrium ratio between these two variables, as determined above in Figure 1. The intersection between the curve F and the line with slope z together determine each of the numbers of the two types of sectors. Once these two numbers are determined, the full equilibrium in the economy is determined with them, through the various equilibrium conditions in Section 3. We can therefore summarize this discussion by the following proposition:

Proposition 2: If the parameter χ is non-positive, there exists a unique equilibrium, which is described by the numbers of sectors (J_M, J_T) and by the threshold s, and this equilibrium is stable.

5. Economic Growth and Diversification

In this section we examine the relationship between economic growth and the distribution of sectors in our model. We do it by analyzing how the economy is affected by changes in the productivity of the modern sectors A. We first turn to Figure 1, which describes how the equilibrium is determined. As noted above, changes in A have no effect on the G curve, but as productivity A increases, the M curve shifts down according to equation (18) and to our assumption that ρ is positive. As a result, both s and z are reduced. Namely, technical progress reduces the threshold between the two types of labor. Hence, more individuals supply efficiency labor and more people work in the modern sectors. At the same time the number of traditional sectors relative to modern sectors decreases.

This result is strongly related to Proposition 1, according to which the substitutability between traditional and modern goods in the economy is higher than 1. As productivity of modern intermediate goods rises, more workers move to these sectors and their number increases on the expense of traditional sectors. This can happen only if there is high substitutability between the two goods. When production of the modern good increases, more sectors are created and the producers of the modern good use a growing number of intermediate goods. This also requires high substitutability between them, since otherwise the shift of workers to modern production will increase each sector, but will not increase much the number of modern sectors. Similarly our story also requires a reduction of traditional sectors along this process of growth and hence the substitutability between traditional intermediate goods also needs to be high.

We next turn to examine how a rise in productivity A affects not only the numbers of each type of sectors, as can be learned from Figure 2, but also the overall concentration or diversification to sectors in the economy. One measure to such diversification is the total number of sectors in the economy, namely $J = J_T + J_M$. The effect of growth on this diversification can be analyzed in a simple diagram, as done in Figure 3. As in Figure 2, the intersection of the Sectors Frontier F and the diagonal at slope z determine the numbers of the two types of sectors. But Figure 3 adds the total number of sectors, which is the intersection of the horizontal axis with a line of slope -1 that passes through the equilibrium point Q.

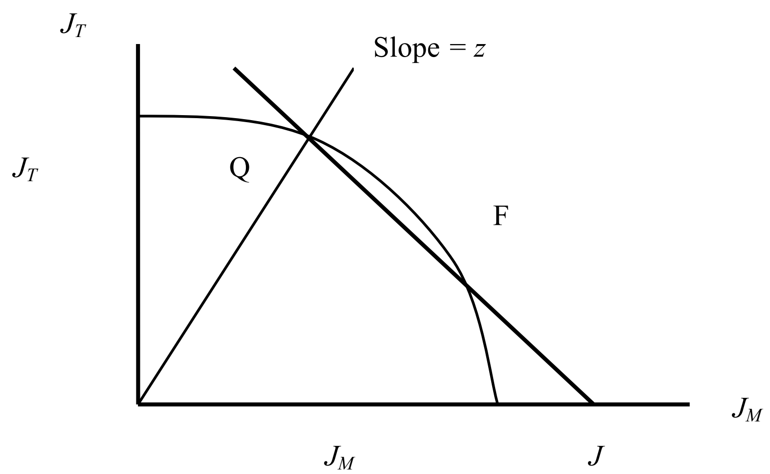

Figure 3

A rise in productivity A reduces z and thus increases J_M and reduces J_T, since it shifts the economy to the right and down on the curve F. That increases the total sum of sectors J since it shifts the line with slope -1 to the right. As Figure 3 shows, at some point this result switches and a further shift down the F curve starts to move this line to the left and therefore reduces the total sum of sectors. This switching occurs exactly where the slope of F is equal to -1, namely when the marginal opportunity cost of building another modern sector is exactly equal to 1. This behavior fits the empirical findings of Imbs and Wasciarg (2003), namely that sector decentralization rises and then declines along the path of economic growth. But this result might not always be reached in our model, as it requires that there is a point on F with a slope of -1, namely inside the Sector Frontier F. This issue is examined in the next proposition.

Proposition 3: The sector frontier F has a point with a slope of -1 between $J_M = 0$ and $J_T = 0$, if the maximum efficiency E satisfies: $E > 1$. Hence, if $E > 1$, diversification, if measured by the total number of sectors J, follows an inverse U shaped path along economic growth, as it first rises and then declines.

Proof: in the Appendix.

The explanation to this result is the following. The amount of employment in each traditional sector is m. The amount of efficiency labor in each modern sector is the same, but that is not the amount of people employed. The average efficiency in modern sectors is $(E+s)/2$. Hence the amount of employed people in such a sector is:

$$\frac{2m}{E+s}.$$

When modern sectors begin to form, s is high and close to E, so that the number of employed in the first modern sector is actually equal to m/E. Hence, the condition $E > 1$ means that the required number of employees in a modern sector at the beginning of economic growth should be smaller than the required amount in a traditional sector. This is of course in general the condition for the curve F to have an internal point with a slope -1.

This is therefore the main result of the paper. It is interesting in itself, as most other theoretical research on this issue has stressed possible monotonic relations between growth and diversification, while this model raises the possibility of a non-monotonic relation. Furthermore, it is a result of a very simple and common assumption in Economics, increasing marginal costs. This result is important also because it fits well the empirical findings of Imbs and Wasciarg (2003). Proposition 3 uses a simple measure of diversification, namely the total number of sectors. We next show that a similar result is obtained also if diversification is measured differently, by the distribution of labor over sectors, as measured by Imbs and Wasciarg (2003). In a similar way, we measure the concentration of sectors by applying to the distribution of labor the Herfindahl index, or the Herfindahl-Hirschman Index (HHI).[13]

[13] The results are similar if we use other measures, like Gini.

Note that concentration is actually an opposite measure to diversification of labor across sectors. The next proposition analyzes the relationship between sector concentration and economic growth using this measure. It shows that the conditions for a U shaped relationship are very similar to those in Proposition 3.

Proposition 4: If $4 > E > 1$, concentration, as measured by HHI of labor shares in sectors, declines as A begins to grow and then rises from some A on.

Proof: In the Appendix.

6. Two Extensions of the Model

This section explores two extensions to the model. One extension assumes that acquiring expertise is costly, but these costs decline with scale. Hence this extension allows for more than one firm in each sector, so we get competition instead of monopoly in each sector. The second extension studies how changes in human capital might affect the equilibrium distribution of sectors, if human capital increases the number of people who can supply efficiency labor and work in modern sectors.

6.1 Sectors with Many Firms

In the benchmark model every sector hosts only one firm. The reason is that every new firm prefers to create a new sector, where the demand price is high, than to join an existing sector and face a lower demand price. Note that the monopolistic competition in this model is not a result of some fixed costs, but a result of incentives for product differentiation. In order to have more firms in each sector we need to introduce a force that counters the incentives to differentiate. We introduce such a mechanism in this sub-section. It is sketched briefly and the full analysis is left to future research.

Assume that the model is the same as the benchmark model, but that sector expertise does not come for free. Learning it requires effort. Assume further that this effort is decreasing with the size of the sector, because people can learn from others. Hence, although revenues from going to a new sector are higher, learning the required know-how is more costly, since the entrants are the first experts in that sector. As a result, new firms prefer to enter an existing sector. We show that if there is a lower bound to the effort of learning, firms will enter an existing sector only up to some size, and then move to a new sector. To demonstrate it, assume that while utility of consumers is equal to *c*, utility of experts is equal to *c/F*, where *F* is their effort of learning the required know-how. Assume further that *F* depends on the number of firms in a sector *n* in the following way:

$$F = f + \frac{1}{n},$$

where $f > 1$. We can justify this assumption in the following way. Some effort *f* is always exerted, while the rest of the effort diminishes with the number of firms producing the same good that surround you. Clearly, the income required by experts in order to justify their effort is Fw_l in traditional sectors and Fw_h in modern sectors.

The revenues of each competitive firm minus its costs of production labor, namely its operating profits, are proportional to its price, which is proportional to $n^{\sigma-1}$, according to demand equation (6) and (7). The cost of experts is proportional to $f + 1/n$, as explained above. Figure 4 draws the two curves, of operating profits and cost of experts as functions of the number of firms in the sector. The figure is drawn for the case that a single firm in the sector has zero profits. Initially the cost function is steeper and hence profits of next entrants to the sector are positive. This goes on until the size of the sector reaches *n** firms, where profit becomes zero again. The next firms will already move to a new sector.

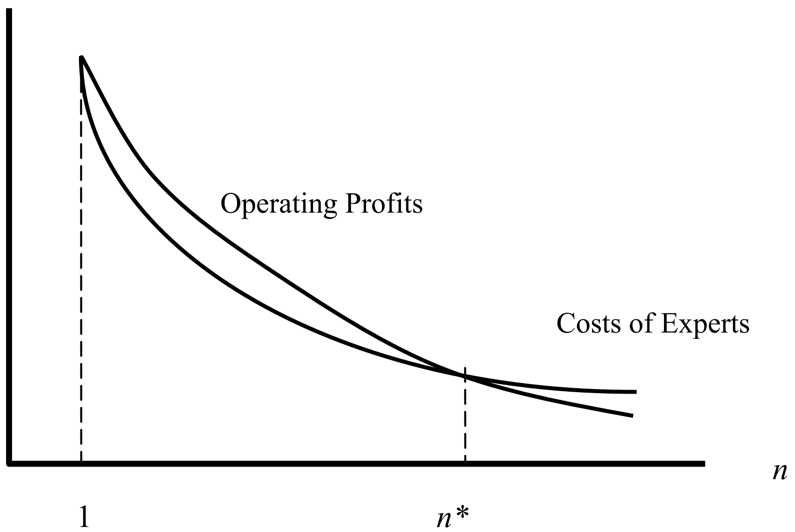

Figure 4

To find the number of firms in a sector, n^*, note that the ratio between $(n^*)^{\sigma-1}$ and $f+1/n^*$ should be the same as the ratio between these two functions at n = 1. Hence, n^* is determined by the following equation:

$$(21) \qquad (n^*)^{\sigma}(f+1) = fn^* + 1.$$

Note that the number of firms in a sector n^* does not depend on productivity A, or on other variables of the model. Thus, the number of firms in a sector in equilibrium is larger than 1 and if this number is sufficiently large there will be perfect competition in the sector. Hence, the assumption of increasing returns to scale in acquisition of sector know-how enables entry of more than one firm to each sector, so that there will be competition instead of monopoly. Note that the number of firms in each sector is independent of the sector and hence so is employment and production in each sector. From this we infer that all the results of the benchmark model, including that diversification follows an inverse U shape as the economy grows, hold in this extension as well. Hence, the main results of the paper do not depend on the type of competition in each sector at all.

6.2 Cross-Country Differences in Human Capital

In this sub-section we extend the model so that it can account not only for differences in productivity A across countries, but also for differences in human capital across countries. Assume that not all workers have efficiency labor, but only those who study and acquire human capital. The number of such people is denoted by H. The others have only raw labor and can work only in the traditional sectors. Human capital in this extension affects the supply of efficiency labor. The equilibrium conditions of this extension are adjusted easily. The efficiency labor market clearing condition yields:

$$(22) \qquad J_M m = \int_s^E e\frac{H}{E}de = \frac{H}{E}\frac{E^2 - s^2}{2}.$$

The raw labor market clearing condition yields:

$$(23) \qquad J_T m = L - H + \int_0^s \frac{H}{E}de = L - H + \frac{H}{E}s.$$

Dividing these two conditions we get that in this extension the G curve is described by:

(24) $$\frac{J_T}{J_M} = 2\frac{EL/H-(E-s)}{E^2-s^2}.$$

The M curve in this extension is the same as in the benchmark model and is described by (18) as well. Finally note that the Sector Frontier in this extension of the model is described by the following equation, which is derived from (22) and (23):

(25) $$J_T = \frac{1}{m}\left[L-H+H\sqrt{1-\frac{2m}{E}\frac{J_M}{H}}\right].$$

Note first, that in this case the number of traditional sectors is bounded from below, since even if the economy reaches maximum production of modern sectors, $L-H$ workers still work in traditional sectors. The maximum amount of modern sectors is $(H/m)(E/2)$. When human capital increases, the Sector Frontier moves outward, but not in parallel, as the maximum number of modern sectors increases while the maximum number of traditional sectors remains unchanged, at L/m.

We next turn to analyze the effect of a rise in human capital H on the economy. First, according to equation (24) such an increase in human capital reduces L/H, so that the G curve shifts to the left. As a result s increases and the ratio of numbers of sectors z declines. This is an interesting result in itself. If the number of people who can supply efficiency labor increases, a smaller share of them goes to the modern sectors. Namely, the increased supply of efficiency labor does not end up fully in modern sectors. As a result, the average ability of efficiency labor, $(E+s)/2$, increases. As for the effect of a rise in human capital on the numbers of sectors, note that in addition to the decline of the ratio z, the Sector Frontier itself shifts outward. This clearly increases the amount of modern sectors J_M. The effect on the number of traditional sectors J_T is less clear. Next we examine the effect of an increase in H on the total number of sectors J. The derivative of J with respect to H is equal to:

(26) $$\frac{dJ}{dH} = \frac{(E-1)^2-(s-1)^2}{2mE} + \frac{H}{mE}(1-s)\frac{ds}{dH}$$

The first element in the derivative is always non negative. The derivative ds/dH is non-negative. We next show that the effect of increase of human capital on the total number of sectors depends on whether the economy is developed or not. Consider first the case of a less developed economy, due to a low A (and could be low H as well). In this case s is very close to E, which makes the first element in the derivative close to zero. But in such equilibrium the M curve intersects G very far to its right, where it is very flat. Hence, the derivative ds/dH is very low as well. Hence, in this case the derivative (26) is close to zero. If on the contrary the economy is developed and s is low and below 1, the derivative (26) is positive. We therefore conclude: If the economy is not developed, an increase in human capital does not have a significant effect on the total number of sectors. If the economy is developed, an increase in human capital increases the total number of sectors.

It therefore seems that an increase in human capital has a different effect on sector diversification than an increase in productivity in our model. This is not surprising. When productivity rises it increases the demand for efficiency labor, while an increase in H increases its supply. But it is important to stress that the historical data, as described in the introduction, implies that most economic growth over the last two centuries has been driven by technical change and not by human capital. Thus, the effect of increasing productivity A still has the main effect on the dynamics of sector diversification.

7. Conclusion

This paper presents a theory of how sectors are endogenously created and what happens to their distribution along the path of economic growth. The main result of the paper is that, as the economy develops, initially sectors become less concentrated, namely economic activity spreads across a larger variety of sectors but there exists a level of development beyond which sectors begin to concentrate again over a smaller variety of sectors. In other words, the number of sectors follows an inverse U shaped curve, which is consistent with the empirical findings of Imbs and Wasciarg (2003).

The paper distinguishes between modern and traditional sectors and builds on a very standard assumption in economics, which is the assumption of increasing marginal costs. Assuming that individuals are equally endowed with raw labor, while efficiency labor is randomly assigned across them, gives rise to increasing marginal opportunity cost of setting up new modern sectors. Since throughout the growth process economies move from traditional to modern sectors and since individuals with more efficiency labor units are employed first, the marginal reduction of traditional sectors increases with the process.

In addition to the result on the dynamics of diversification our paper also finds some restrictions that are required to guarantee the stability of equilibrium and to guarantee that it is well-behaved, when variables change. These restrictions on the parameters of the model have interesting implications with respect to the elasticities of substitution in the economy. We show that these restrictions imply that the elasticity of substitution between the modern and traditional goods should be greater than 1, and that the elasticity of substitution between traditional goods themselves and between modern goods themselves should be even higher than that. These results are interesting and seem to deserve some additional research, both theoretical and empirical.

Another potential research that this paper might lead to is extending the empirical study of Imbs and Wasciarg (2003) to distinguish between traditional and modern sectors. Our model implies that the relation between the numbers of the two types of sectors should form a concave frontier. Our model also implies that changes in human capital might shift this frontier. Such an empirical analysis can benefit from the recent contribution of Caselli and Coleman II (2006), which measures for each country the productivity of high-skilled and of low-skilled labor and shows that these two variables are related across countries by a concave frontier. They also show that this frontier shifts over groups of countries. This similarity can help us in conducting such an empirical analysis, which could give further support to our model on sector dynamics.

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Appendix

1. Discussion of the Case of positive χ: Assume that χ >0. From equations (17) and (18) we can derive the following equilibrium condition:

$$\text{(A.1)} \qquad s = \left(\frac{2s}{E^2 - s^2} \right)^{\chi} A^{-\rho}.$$

The equilibrium s is equal to the ratio of real wages, w_l / w_h, which is the right hand side of (A.1). Hence, if s is above the wage ratio, s declines, while if it is below the wage ratio, s rises. To analyze equation (A.1) diagrammatically, we rewrite it slightly differently:

$$\text{(A.2)} \qquad sA^{\rho} = \left(\frac{2s}{E^2 - s^2} \right)^{\chi}.$$

As discussed above, as long as the LHS of (A.2) is above the RHS, s declines, and if the LHS is below the RHS, s rises.

Note, that the LHS of (A.2) is a linear line from the origin with a slope A^{ρ}. As for the RHS, distinguish between two cases, $\chi < 1$, and $\chi \geq 1$. The RHS is an increasing function of s. As s approaches E, the RHS of (A.2) approaches infinity. The derivative of the RHS of (A.2) with respect to s is:

$$2\chi \left(\frac{2s}{E^2 - s^2} \right)^{\chi - 1} \frac{E^2 + s^2}{\left(E^2 - s^2\right)^2}.$$

Hence, if $\chi \geq 1$, the RHS is convex. If $\chi < 1$, the RHS is first concave, as its derivative at s = 0 is infinity, and then it becomes convex at some point, since it should rise to infinity as s approaches E. The following diagram presents the LHS and the RHS in the two cases and their intersections. Note that all RHS curves reach the value 1 at the same value of s, $s^* = \sqrt{1 + E^2} - 1$. All the curves are presented in Figure A.1.

If χ is greater than or equal to 1, the RHS is convex and there are two points of intersection, O and B. According to the dynamic analysis above, B is unstable. O is a stable equilibrium, but it has a problematic economic implication, namely, that for any level of productivity A, even if it is very low, s is equal to zero so that all workers supply efficiency labor and there are only modern sectors. We therefore should rule out this case.

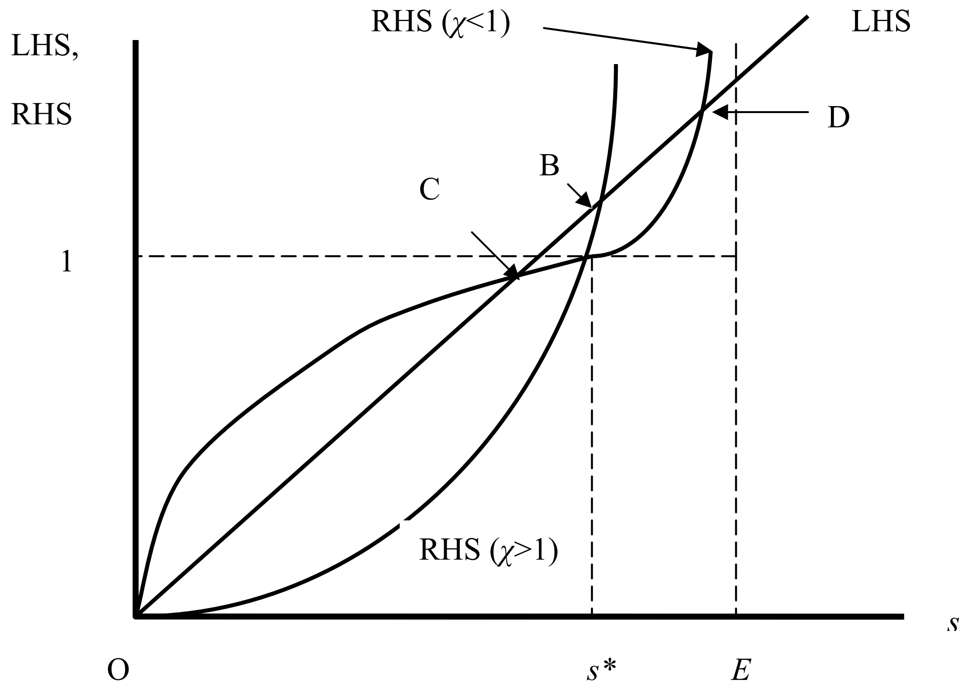

Figure A.1

If χ is smaller than 1, there are three equilibrium points, D, C and O. D and O are unstable. The only stable equilibrium is C. Consider next the effect of reducing productivity A. It shifts the LHS curve down. The reason is that if χ is positive and if we also assume that σ is positive then ρ should be positive as well. As A is reduced, the LHS reaches a point where it is completely below the RHS. In this case s jumps up all the way to E, so there is no production in the modern sectors. This means that the equilibrium in this case has a point of non-continuity with productivity A.

2. Proof of Proposition 3: The slope of the sector frontier, which is actually the marginal cost of setting a new modern sector, is simply the derivative of (20), which is:

$$\frac{\partial J_T}{\partial J_M} = -\frac{1}{E}\left[1 - \frac{2m}{EL} J_M\right]^{-\frac{1}{2}}.$$

At $J_M = 0$, the derivative is equal to:

$$\left.\frac{\partial J_T}{\partial J_M}\right|_{J_M=0} = -\frac{1}{E}.$$

This slope is higher than -1 if $E > 1$. At $J_T = 0$ the derivative is minus infinity. QED.

3. Proof of Proposition 4: The number of workers and experts in each traditional sector is m, while the number of workers and experts in each modern sector is $2m/(E+s)$. Hence the Herfindahl index of the economy is:

$$\text{HHI} = \sum_{j=1}^{J} s_j^2 = J_T\left(\frac{m}{L}\right)^2 + J_M\left(\frac{2m}{L(E+s)}\right)^2,$$

where s_j is the share of labor in sector j. Substituting the numbers of sectors of each type from equations (15) and (16) we get after some manipulations:

$$\text{HHI} = \frac{m}{LE}\left(s + 2\frac{E-s}{E+s}\right).$$

Hence, the derivative of the Herfindahl index with respect to s is:

$$\frac{\partial(\text{HHI})}{\partial s} = \frac{m}{LE}\left[1 - \frac{4E}{(E+s)^2}\right].$$

As economic growth increases and A grows from 0 on, s declines gradually from E to 0. In the beginning of economic growth s is equal to E and thus the derivative is equal to:

$$\left.\frac{\partial(\text{HHI})}{\partial s}\right|_{s=E} = \frac{m}{LE}\left[1 - \frac{1}{E}\right].$$

Hence, if $E > 1$ the derivative is positive, HHI rises with s and falls with A. Namely, at the beginning of economic growth sector concentration declines. When A is very high s is close to 0 and there the derivative is:

$$\left.\frac{\partial(\text{HHI})}{\partial s}\right|_{s=0} = \frac{m}{LE}\left[1 - \frac{4}{E}\right].$$

Hence, if $E < 4$ the derivative is negative and concentration declines with s, which means that it rises with A. Hence from some point on, the concentration of sectors rises with economic growth. QED.